GENERALIZED MULTILAYER ANISOTROPIC DIELECTRIC RESONATORS

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ABSTRACT

Modeling of the generalized multilayer cylindrical anisotropic dielectric loaded resonator structure by rigorous mode matching method is presented. Eigen modes of the multilayers two parallel plates waveguides are obtained. By cascading the radial discontinuities of the structure, resonant frequency, field distribution and the unloaded Q of the resonator are obtained. The computed results are compared with the experimental data for higher order Whispering Gallery (WG) modes and shown to be in good agreement .

I. INTRODUCTION

With the breakthrough of the ceramic technology, a number of new high dielectric constant materials with high quality factor, low temperature coefficient were developed. Tremendous progress on stabilization and miniaturization of cavity resonators and filters has been achieved in the past three decades. Dielectric loaded resonators and filters with high unloaded Q have been widely used in the communication systems and other microwave applications [1]-[5].

Recently, cooled, ultra-high Q, high stability sapphire dielectric resonators found important applications in construction of ultra-stable low noise microwave oscillators [5]-[9]. Sapphire resonator operating at whispering gallery (WG) mode (hybrid mode with large number of azimuthal variations) can achieve extremely high value of Q factor at X-band at liquid nitrogen to liquid helium temperatures without using superconductors. As single crystalline sapphire is a dielectric with uniaxial anisotropy, the influence of the anisotropic dielectric constants on the resonant modes of the resonator has to be taken into account in the resonator design. The design of such resonators requires the accurate computation of the resonant frequencies and the unloaded Q of both spurious and operating modes.

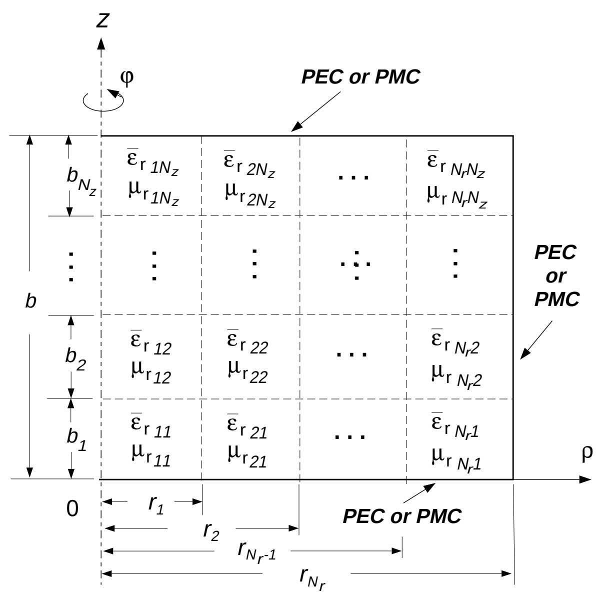

Fig. 1. Configuration of the generalized multilayer cylindrical uniaxial-anisotropic dielectric loaded resonator

Few analysis methods have been reported for the study of uniaxial-anisotropic dielectric resonators. Although rigorous mode matching techniques in [6][10] are used to solve the solid and ring type sapphire resonators, only the resonant frequencies of the resonators are obtained. Other important parameters of the resonator, such as field distributions, unloaded Q spurious free window WG modes, and the frequency sensitivity of the anisotropic resonator, are not seen in the literature. Furthermore, the effects of the support structure on the resonator are needed to be taken into account in the resonator design. Thus a modeling method for analysis of the generalized uniaxial anisotropic multilayer dielectric loaded resonator structure needs to be developed for design of the sapphire and other multilayer dielectric loaded resonators and filters.

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In this paper, the modeling of the generalized multilayer cylindrical anisotropic dielectric loaded resonator structure by rigorous radial mode matching method is presented. Eigen modes of the generalized multilayer two parallel plates waveguides are obtained. By cascading all the radial discontinuities of the structure, resonant frequency, field distribution, and unloaded Q of the resonator are obtained for any mode. The frequency sensitivity to the enclosure dimension changes of the resonator can be accurately determined by using the perturbation theory. The correctness of the theory is verified by comparing the computed results with the measured results.

II. CONFIGURATION AND THEORY

The configuration of a generalized multilayer cylindrical uniaxial-anisotropic dielectric loaded resonator under consideration is shown in Fig. 1, in a cylindrical coordinate system (ρ, ϕ, z). There are N_z layers in z-direction, and N_r layers in r-direction. Therefore, the structure can be partitioned into $N_z \times N_r$ regions to be able to be analyzed by mode matching method. Each region can be filled with a uniaxial-anisotropic dielectric material with relative permittivity tensor $(\bar{\epsilon}_r)_{ij}$, loss tangent $(tan\delta)_{ij}$, and relative isotropic permeability $(\mu_r)_{ij}$. The top, bottom and side wall can be either perfect electric conductor (PEC) or perfect magnetic conductor (PMC).

The permittivity tensor $\bar{\epsilon}_r$ is given by:

$$\bar{\epsilon}_r = \begin{bmatrix} \epsilon_t & 0 & 0 \\ 0 & \epsilon_t & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \qquad (1)$$

For isotropic case, the dielectric constants ϵ_t and ϵ_z are equal. Since the number of layers N_z, N_r of the resonator are arbitrary and the dielectric constant in each region can also be chosen arbitrarily, the configuration is very general and allows almost unlimited types of structures to be analyzed.

Starting from the Maxwell's equations, the wave equations in the charge free uniaxial anisotropic medium can be obtained as:

$$\nabla^2 \vec{E} - \left(1 - \frac{\epsilon_z}{\epsilon_t}\right) \nabla \left(\frac{\partial E_z}{\partial z}\right) + k_o^2 \mu_r \epsilon_z \vec{E} = 0 \qquad (2)$$

$$\nabla^2 \vec{H} - j\omega\epsilon_o\epsilon_t \left(1 - \frac{\epsilon_z}{\epsilon_t}\right) \nabla \times (\hat{z} E_z) + k_o^2 \mu_r \epsilon_t \vec{H} = 0 \quad (3)$$

Since the components of $\vec{E}$ and $\vec{H}$ are not all independent, it is not necessary to solve all six scalar wave equations for the six field components at the same time. To

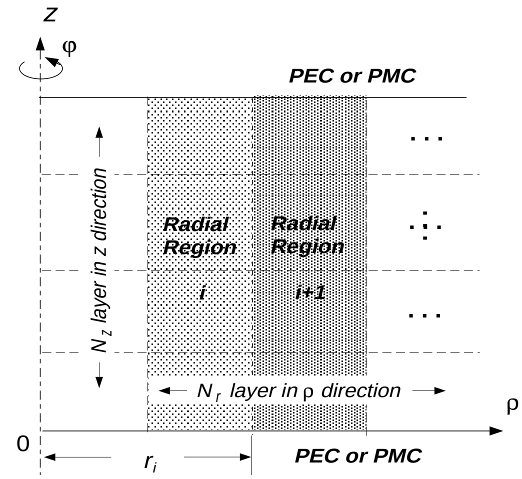

Fig. 2. Multilayer two parallel waveguides and radial discontinuity in the dielectric loaded resonator

simplify the analysis, it is usual to decompose the normal mode fields into two orthogonal sets of solutions, *i.e.* TE_z modes ($E_z = 0$) and TM_z modes ($H_z = 0$). The transverse field components can then be expressed in term of the E_z and H_z fields.

Rigorous radial mode matching method is then applied to model the resonator. In this method, the resonator is divided into N_r regions along ρ direction, according to its radial discontinuities, and each radial region has N_z layers in z direction, as shown in Fig. 2. From the top and bottom plate's boundary conditions of the two parallel plates radial waveguide, the field expressions of the eigen function in each layer can be obtained. Applying the boundary conditions at the interfaces between two layers, the characteristic equation for the radial propagation constant of the eigen modes can be obtained by repeatedly computing the field coefficients of the next layer. The propagation constant of the modes can then be found by searching for the propagation constant which satisfies the characteristic equation. All the field coefficients of the two parallel plates waveguide's eigen function can then be computed.

Having obtained the eigen functions of the two parallel plates waveguide of each radial region, the total electromagnetic fields in each radial region can then be expanded as the linear combinations of the eigen fields of the two parallel plates waveguide. By forcing the tangential electromagnetic fields at the interface between two radial regions, and taking the proper inner product, the continuity equations relating the field coefficients between the two regions can be obtained. Consider the discontinuity between radial region i and region $i+1$, and assuming that the field coefficient relation matrix

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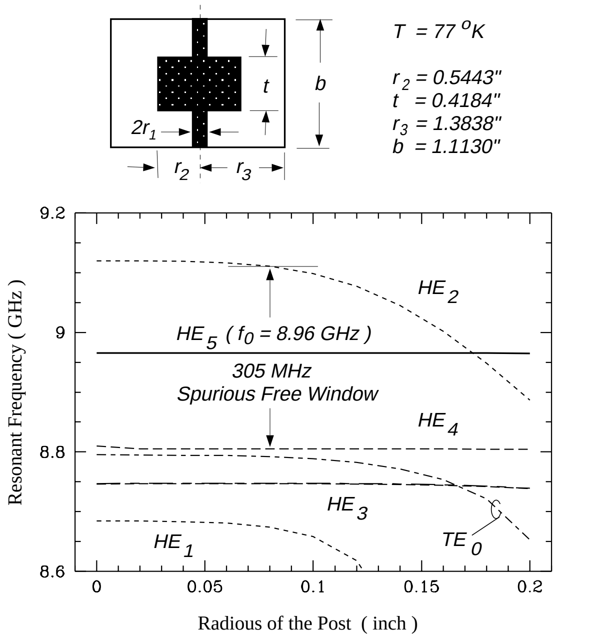

Fig. 3. Mode chart of a solid type HE_5 sapphire resonator as a function of the radius of support

$[T^i_{CD}]$ of the inner region i has been known from solving the discontinuity of the previous region, as:

$$\left[D^i\right] = \left[T^i_{CD}\right]\left[C^i\right] \tag{4}$$

Substitute it into the continuity equations, the field coefficient relation matrix of the outer region $i+1$ can then be obtained:

$$\left[D^{i+1}\right] = \left[T^{i+1}_{CD}\right]\left[C^{i+1}\right] \tag{5}$$

Since all the elements of the field coefficient matrix $\left[T^1_{CD}\right]$ of the inner most region are zero, an equation relating the coefficients of the outer most region can be obtained by repeatedly cascading the coefficient matrix from inside to outside of the resonator. Applying the boundary conditions at the side enclosure wall, the characteristic equation for the resonant frequency of the resonator can finally be obtained. Searching for the zero determinant of the characteristic equation gives the resonant frequencies of the resonant modes of the analyzed uniaxial anisotropic dielectric loaded resonator. The field coefficients of the resonant mode in each region can then be obtained by solving the characteristic and continuity equations.

The computation of the unloaded Q involves the calculation of the stored energy $W_{E,H}$ of the resonant mode in the structure, dielectric loss P_d, and the conductor losses P_c at the enclosure. Since all the eigen mode functions and their field coefficients are known,

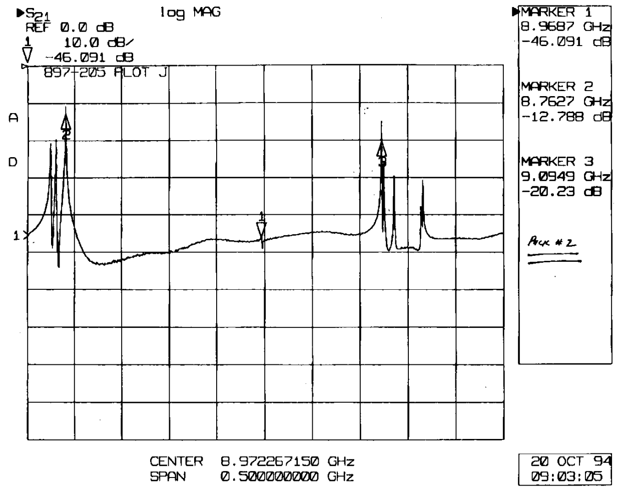

Fig. 4. Measured resonant frequency and spurious free window of the HE_5 resonator

the above computation can be achieved analytically, which yields high computational efficiency and accuracy especially for WGM resonators. The total unloaded Q of the resonator is computed from:

$$\frac{1}{Q_u} = \frac{1}{Q_d} + \frac{1}{Q_c} = \frac{1}{\omega_o \frac{W_{E,H}}{P_d}} + \frac{1}{\omega_o \frac{W_{E,H}}{P_c}} \tag{6}$$

The separation of Q_d and Q_c helps to understand the loss mechanism of the structure and to optimize the dimensions of the resonator.

III. NUMERICAL RESULTS

A computer program has been developed to compute the resonant frequency, field distribution, unloaded Q and frequency sensitivity of the cylindrical multilayer uniaxial anisotropic dielectric loaded resonators. As the resonant frequencies of most low order modes are quite sensitive to either the height of the enclosure, or to the radius of the WGM resonator, only at certain dimension of the height and radius of the enclosure, balanced large spurious free window can be obtained. Fig. 3 shows the mode chart of an optimized solid HE_5 sapphire resonator with the support as a function of the support radius. It is seen that the resonant frequencies of the HE_5 mode is nearly invariable to the change of the support radius, which implies that the resonator is highly stable. The optimized post radius is found to be at 0.075" which gives 305 MHz spurious free window. The measured results of the resonant frequency and the spurious free window of the resonator with 0.075" supporting post is shown in Fig. 4. The marker 1 shows the resonant frequency of the HE_5 mode, and marker

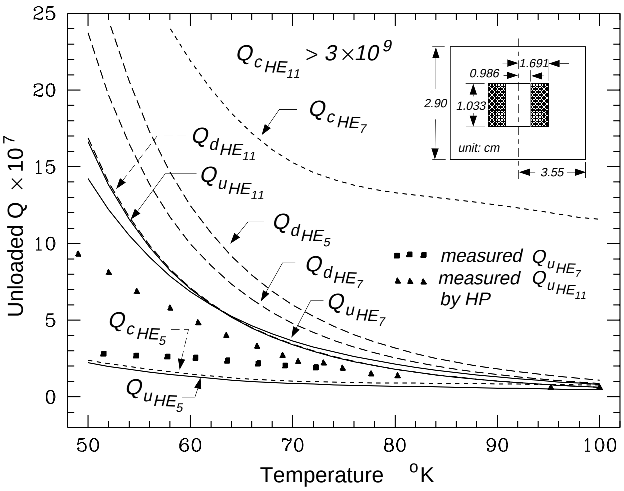

Fig. 5. Contributions of the dielectric and conductor unloaded Q of a sapphire resonator as function of temperature

2 and 3 indicate the resonant frequencies of HE_4 and HE_2 modes, corresponding to the resonant modes in Fig. 3 at $r_1 = 0.075"$, respectively. The excellent agreement between the computed and measured results again shows the correctness of the theory and accuracy of the results.

Fig. 5 shows the typical contributions of the dielectric loss and conductive losses of a ring type WGM sapphire resonator operating at HE_5, HE_7 and HE_{11} modes as a function of temperature between 50 and 100 K. The resonant frequencies of the WGM modes are 8.1, 9.6 and 13.0 GHz, respectively. It is seen that the unloaded Q of the HE_7 and HE_{11} modes are mostly determined by the dielectric loss of the resonator, while the conductive loss is dominant for HE_5 mode. The computed unloaded Qs are compared with the measured results by Flory [6]. The unloaded Q of the HE_{11} mode are close to that by the experiment. But the measured Q of the HE_7 mode are too low which is probably due to the strong influence of some extrinsic loss factors [6], such as poor contact of the enclosure.

IV. CONCLUSIONS

Generalized multilayer cylindrical anisotropic dielectric loaded resonator is modeled by rigorous mode matching method. Resonant frequency, field distribution, unloaded Q of the resonant mode are obtained. The correctness of the theory and accuracy of the results are verified by comparing the measured results.

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